

# Licia Verde

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<http://icc.ub.edu/~liciaverde>

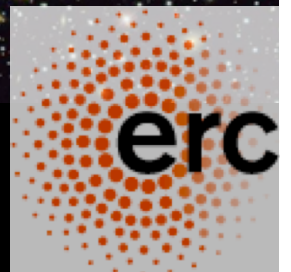
General non-Gaussian shapes in large-scale structure

Wagner, Verde, Boubekur, Paper I: [arXiv:1006.5793](https://arxiv.org/abs/1006.5793) (JCAP 2010)

Wagner, Verde Paper II: [arXiv:1102.3229](https://arxiv.org/abs/1102.3229)



Institut de Ciències  
del Cosmos



# shapes

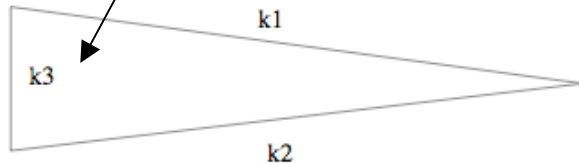
Simple inflationary model:

One field, canonical kinetic energy, slow roll, Bunch-Davies vacuum

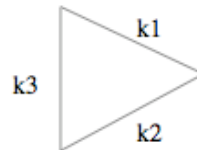
➡ Small LOCAL non-Gaussianity  $\Phi = \phi + f_{NL}(\phi^2 - \langle \phi^2 \rangle)$

Look at bispectrum

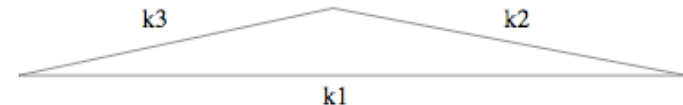
(a) Squeezed



(b) Equilateral



(c) Flattened/Folded



Violation of each of the above conditions leaves a unique signal with specific **shape**

From Komatsu et al. 2009, arxiv:0902.4759 & refs. there



“Non-dog is my co-pilot”

# Tools:

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**CMB:** bispectrum, Topology

**Large-scale structure:**

Bispectrum (or higher orders)

Clustering of peaks on large scales

new

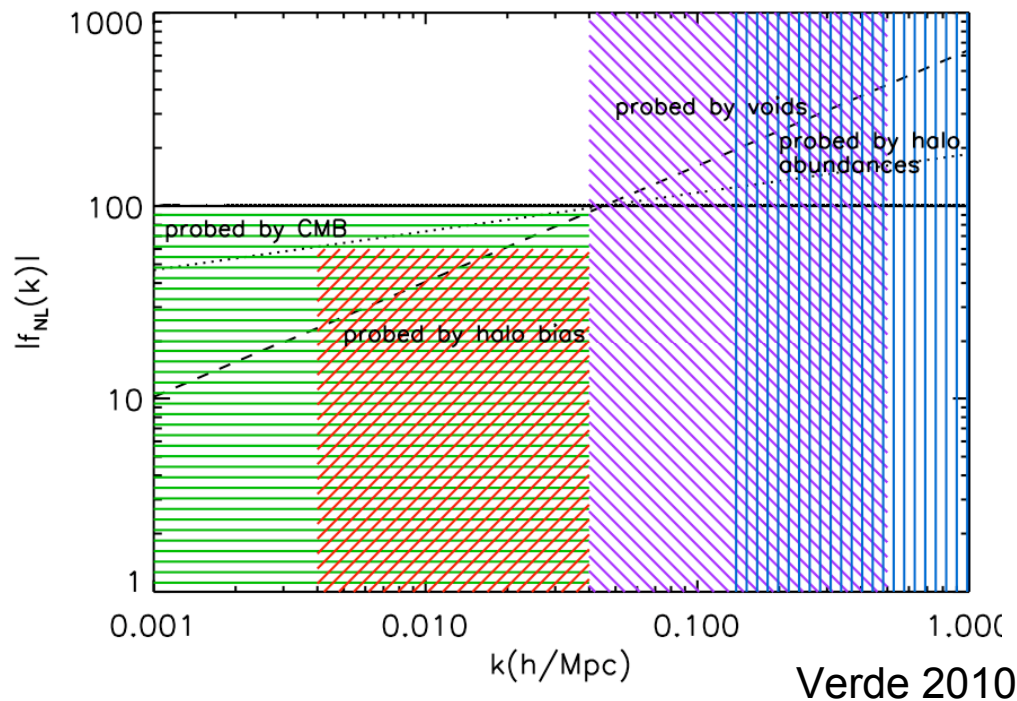
Abundance of rare events (peaks, massive halos...)

(Topology)

Non-linearities always present in LSS: N-body simulations

# Searching for non-Gaussianity with LSS: COMPLEMENTARITY

Each probe is affected by different systematics



In many cases the interpretation gets dirty and messy,  
anyway interesting: can probe smaller scales than CMB

# Non-Gaussian halo bias

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- A Gaussian field and a non-Gaussian field can have the same  $P(k)$
- In a Gaussian field the  $P(k)$  of peaks is completely specified by the  $P(k)$
- In a non-Gaussian field, however, the  $P(k)$  of the **peaks**, depends on all higher order correlations (i.e.  $f_{NL}$ )

# Non-Gaussian halo bias

---

- Gaussian IC and a non-Gaussian IC can have the same  $P(k)$  for the dark matter
- For Gaussian IC the  $P(k)$  of massive halos is completely specified by the dark matter  $P(k)$
- For Non Gaussian IC, however, the  $P(k)$  of the halos, depends on all higher order correlations (i.e.  $f_{NL}$ )

# Non-Gaussian halo bias

For Gaussian initial conditions (known since the '80)

$$\xi_{h,M}(r) = \exp \left[ \frac{\nu^2}{\sigma_R^2} \xi_R(r) \right] - 1 \simeq \frac{\nu^2}{\sigma_R^2} \xi_R(r) \quad b_E = 1 + b_L \quad \text{“The Kaiser formula”}$$

In the '90 this was improved (e.g. Mo & White 1996, Catelan et al 1998)

## For Non-Gaussian initial conditions

Dalal et al. PRD 2008 7713514

Matarrese, Verde, ApJLett, 2008, 77:L77

Slosar et al 08

McDonald 08

Afshordi & Tolley 08

Valageas 2009 etc. etc.

A scale-dependent bias!  
(on top of the Gaussian one  
and proportional to it)

# The Effect

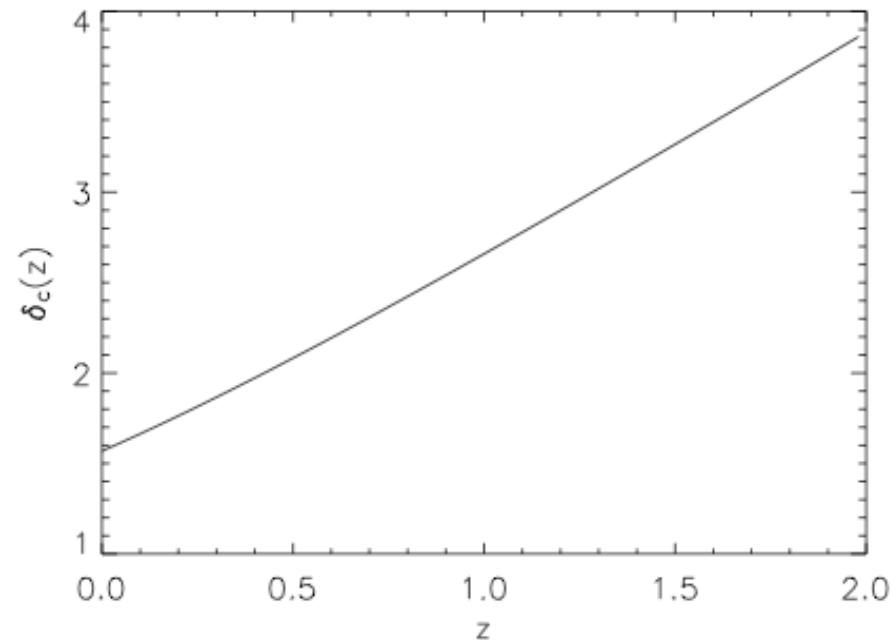
$$\frac{\Delta b}{b^{L,G}} = \beta(k) \frac{\Delta_c(z)}{D(z)}$$

$$\delta(k) = \mathcal{M}_R(k) \Phi(k)$$

$$\alpha = k_1^2 + k^2 + 2k_1 k \mu$$

$$\beta(k) = \frac{1}{8\pi^2 \sigma_R^2 \mathcal{M}_R(k)} \int dk_1 k_1^2 \mathcal{M}_R(k_1) \int_{-1}^1 d\mu \mathcal{M}_R(\sqrt{\alpha}) \frac{B_\phi(k_1, \sqrt{\alpha}, k)}{P_\phi(k)}.$$

Redshift dependence





# The Effect

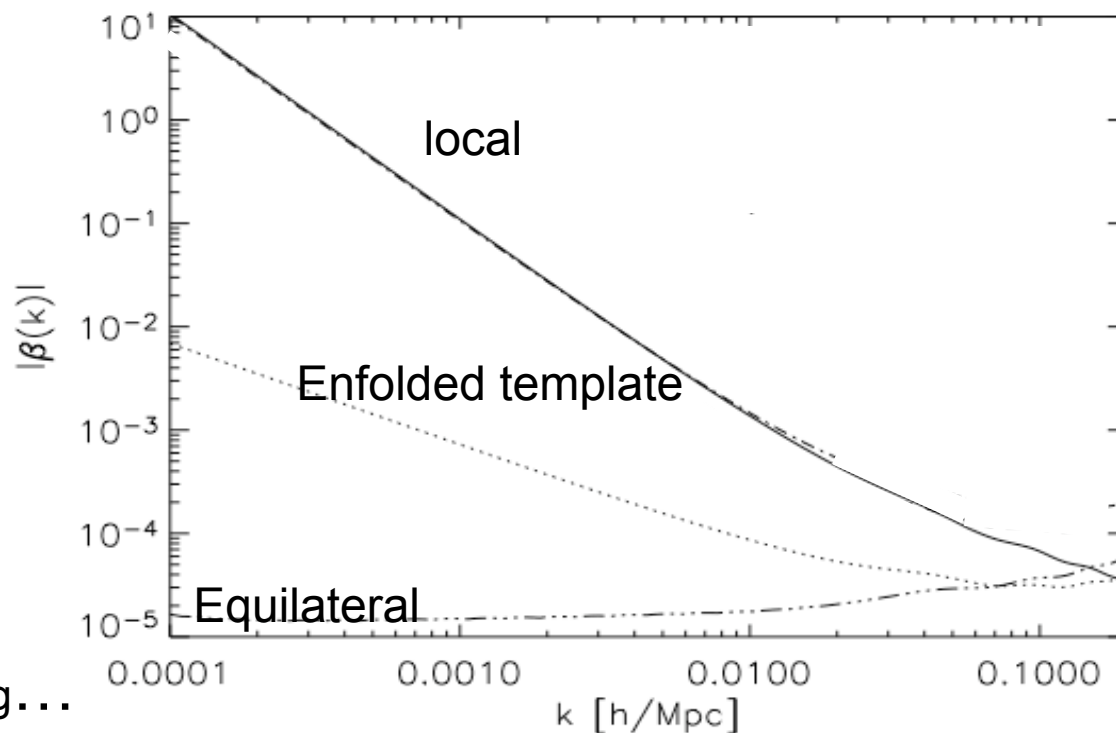
$$\frac{\Delta b}{b^{L,G}} = \beta(k) \frac{\Delta_c(z)}{D(z)} \quad \delta(k) = \mathcal{M}_R(k) \Phi(k) \quad \alpha = k_1^2 + k^2 + 2k_1 k \mu$$

$$\beta(k) = \frac{1}{8\pi^2 \sigma_R^2 \mathcal{M}_R(k)} \int dk_1 k_1^2 \mathcal{M}_R(k_1) \int_{-1}^1 d\mu \mathcal{M}_R(\sqrt{\alpha}) \frac{B_\phi(k_1, \sqrt{\alpha}, k)}{P_\phi(k)}$$

## Scale-dependence

Matarrese, Verde 08  
 Verde, Matarrese 09  
 Taruya et al 08

Interesting...



# How well can this do? Local

Data/method	$\Delta f_{\text{NL}} (1 - \sigma)$	reference
BOSS-bias	18	Carbone et al 2008
ADEPT/Euclid-bias	1.5	Carbone et al 2008
PANNStarrs -bias	3.5	Carbone et al 2008
LSST-bias	0.7	Carbone et al 2008
LSST-ISW	7	Afshordi& Tolley 2008
BOSS-bispectrum	35	Sefusatti & Komatsu 2008
ADEPT/Euclid -bispectrum	3.6	Sefusatti & Komatsu 2008
Planck-Bispectrum	3	Yadav et al . 2007
BPOL-Bispectrum	2	Yadav et al . 2007

Carbone, Mena, Verde 2010:

there is no much degeneracy with cosmology!

Carbone, Verde, Matarrese 08

# Inflationary-GR Intrinsic to LSS

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Bartolo, Matarrese, Riotto 2005, Bartolo et al 2006

Pillepich, Porciani, Matarrese, 2007

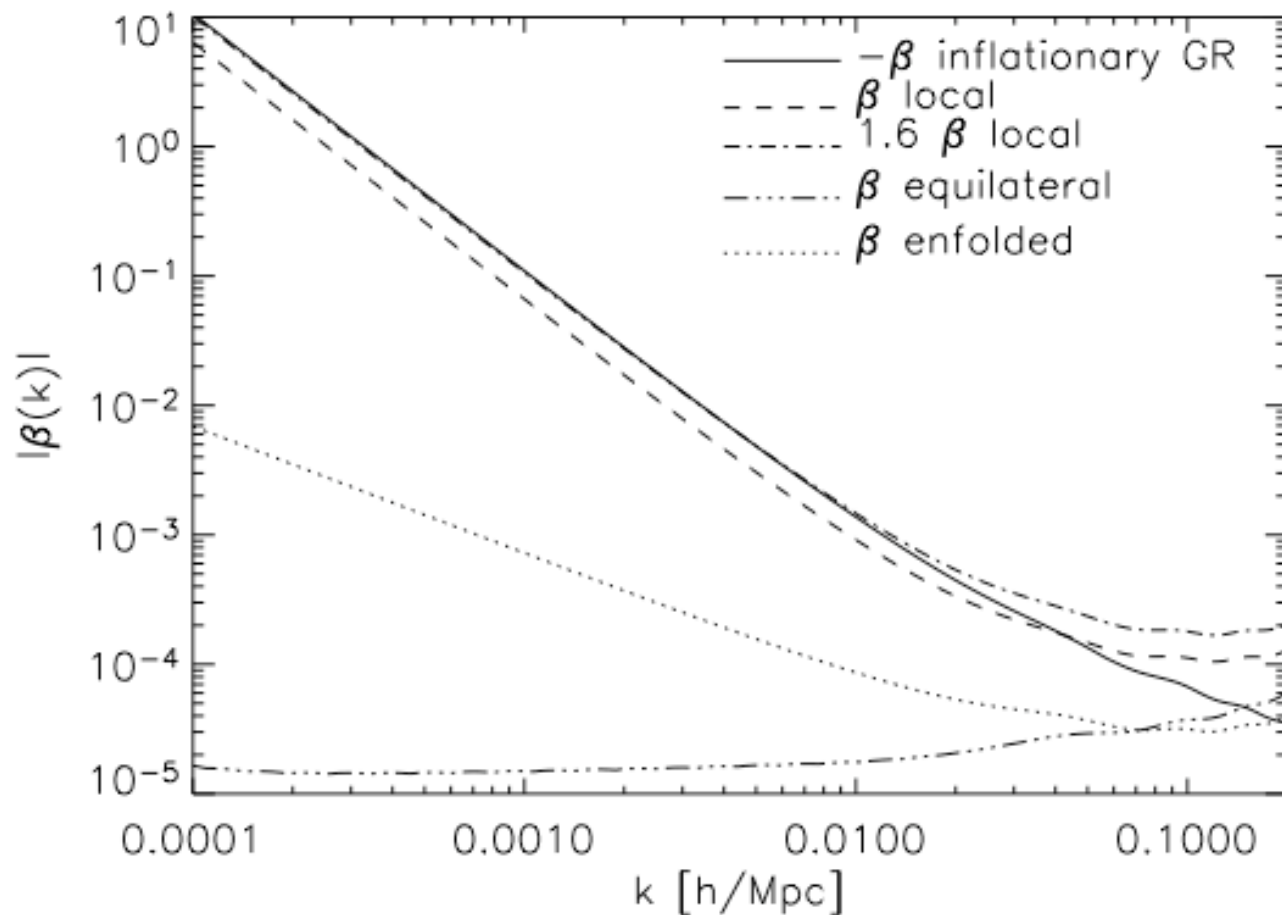
$$B_{\Phi}(k_1, k_2, k_3) = 2 \left[ \frac{5}{3}(a_{\text{NL}} - 1) + f_{\text{NL}}^{\text{infl,GR}}(k_1, k_2, k_3) \right] P(k_1)P(k_2) + \text{cyc.}$$

$$f_{\text{NL}}^{\text{infl,GR}}(k_i, k_j, k_k) = -\frac{5}{3} \left[ 1 - \frac{5 k_i k_j \cos\theta_{ij}}{2 k_k^2} \right]$$

On horizon-scales Poisson equation gets quadratic corrections:  
Needs IC set up of inflation, parallels the TE anti-correlation.

Verde & Matarrese 2009, ApJL

# Inflationary-GR Intrinsic to LSS



On horizon-scales Poisson equation gets quadratic corrections:  
Needs IC set up of inflation, parallels the TE anti-correlation.

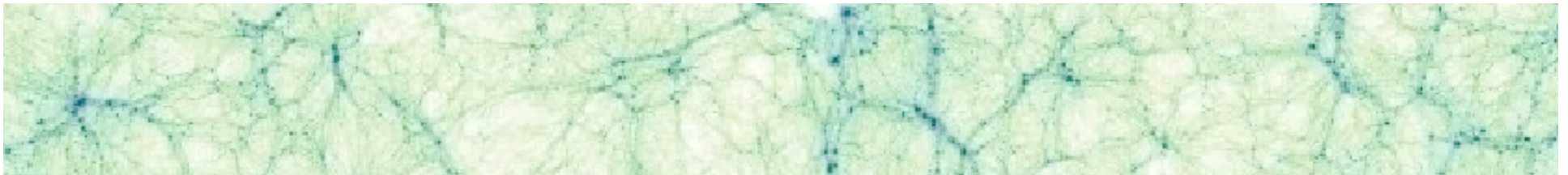
Verde & Matarrese 2009, ApJL

At a potentially detectable level!

# N-body Simulations

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- Halo formation is a highly non-linear process => N-body simulations
- Analytic predictions have been tested with N-body simulations by many groups
- Papers:
  - Dalal et al. 2008
  - Grossi et al. 2007 and 2010
  - Desjacques et al. 2009
  - Pillepich et al. 2010
  - ...
- But up to very recently **only the local type** was simulated!



# Need a big computer and somebody that can use it!



Christian Wagner

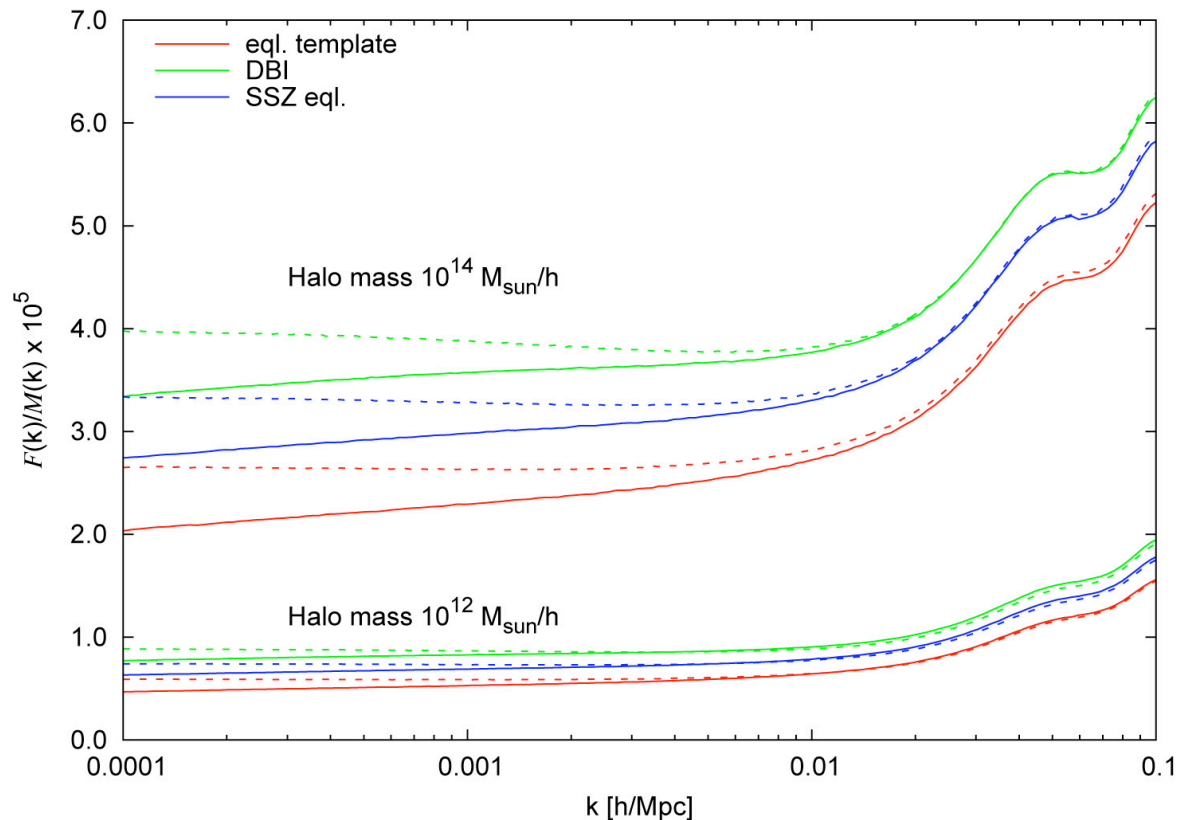
& a lieutenant of the navy  
to admin the machine and  
tune software vs hardware



Hipatia



# Templates vs. physical shapes



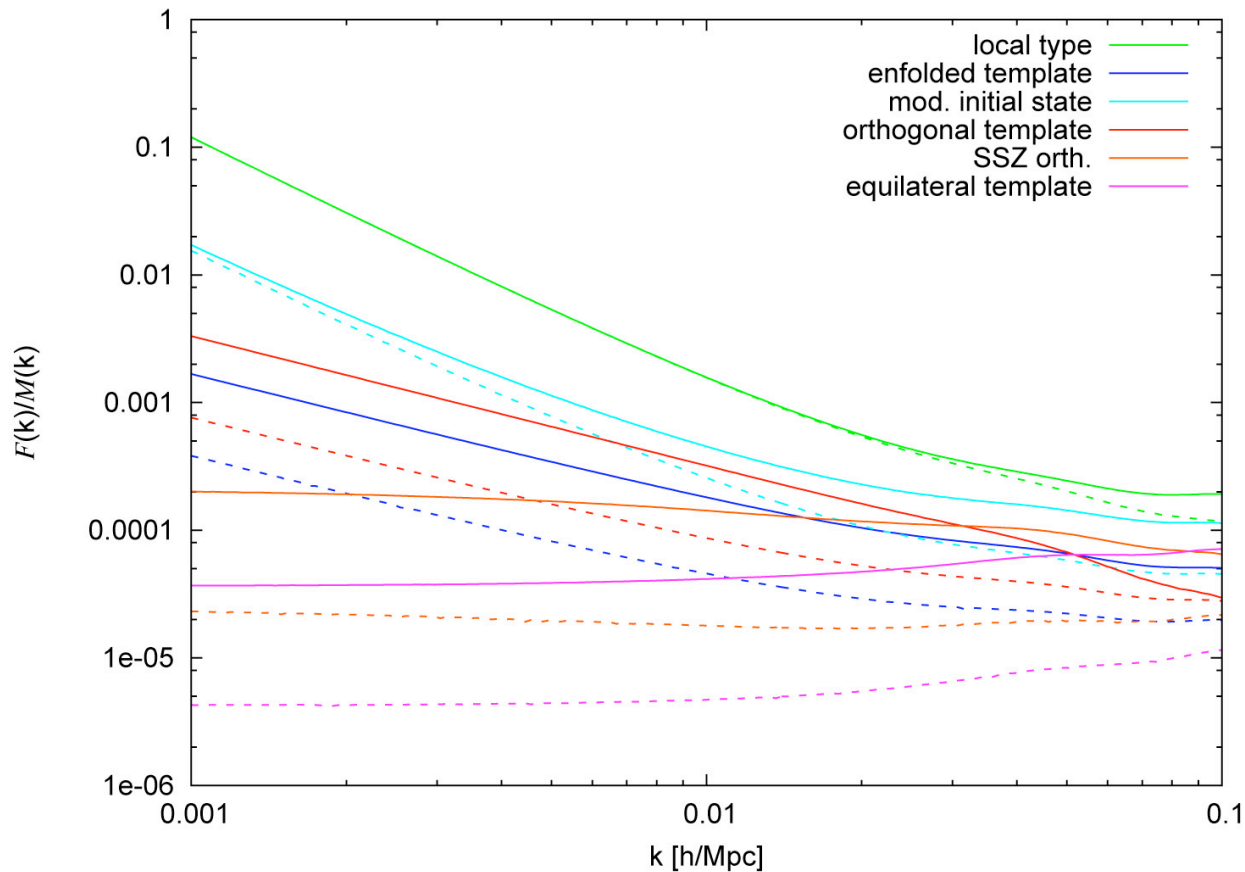
- **templates** approximate the physical bispectra over all triangle configurations and are **factorizable**  
=> allows efficient computation

- however, for the **NG bias** the correct scaling in the **squeezed limit** is crucial

**dashed lines:** scale-invariant power spectrum

**solid lines:** spectral index  $n_s=0.95$

# Templates vs. physical shapes



**solid lines:**  $M_{\text{halo}} \sim 10^{14} M_{\text{sun}}/h$

**dashed lines:**  $M_{\text{halo}} \sim 10^{11} M_{\text{sun}}/h$

- modified initial state/enfolded template (Meerburg et al. 2009)

- orthogonal template (Senatore et al. 2010)

- these templates do **not** have the correct scaling in the squeezed limit

What are the possible squeezed limit scalings?



# Initial Conditions

---

- Split the Potential into a Gaussian and a (small) non-Gaussian part

$$\Phi_{\mathbf{k}} = \Phi_{\mathbf{k}}^G + \Phi_{\mathbf{k}}^{NG}$$

- Generate a Gaussian Random field

$$\Phi_{\mathbf{k}}^G \sim N\{0, (P(k)/2)^{1/2}\} + i N\{0, (P(k)/2)^{1/2}\}$$

$$P(k) = A k^{n-4} \text{ where } n \text{ is the spectral index}$$

- Add  $\Phi_{\mathbf{k}}^{NG}$

- Poisson equation and CMB physics

$$\delta_{\mathbf{k}} = \frac{2 k^2 T(k) D(z)}{3 \Omega_m H_0^2} \Phi_{\mathbf{k}}$$

- Use Zel'dovich Approximation or 2LPT to obtain particle positions and velocities

# How to get $\phi^{NG}$

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- Ansatz for  $\Phi^{NG}$  for a given bispectrum

old (Paper I):

$$\Phi_{\mathbf{k}}^{NG} = \frac{1}{6(2\pi)^3} \int d^3 k_2 B_{\Phi}(k, k_2, |\mathbf{k} + \mathbf{k}_2|) \frac{\Phi_{\mathbf{k}_2}^{*G}}{P(k_2)} \frac{\Phi_{\mathbf{k}+\mathbf{k}_2}^G}{P(|\mathbf{k} + \mathbf{k}_2|)}$$

new (Paper II):

$$\Phi_{\mathbf{k}}^{NG} = \frac{1}{2(2\pi)^3} \int d^3 k' \frac{B_{\Phi}(k, k', |\mathbf{k} + \mathbf{k}'|) \Phi_{\mathbf{k}'}^{*G} \Phi_{\mathbf{k}+\mathbf{k}'}^G}{P_{\Phi}(k)P_{\Phi}(k') + P_{\Phi}(k')P_{\Phi}(|\mathbf{k} + \mathbf{k}'|) + P_{\Phi}(k)P_{\Phi}(|\mathbf{k} + \mathbf{k}'|)}$$

- In both cases:

$$\langle \Phi_{k_1}^G \Phi_{k_2}^G \Phi_{k_3}^{NG} \rangle = \frac{1}{3} (2\pi)^3 B(k_1, k_2, k_3) \delta^D(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)$$

- The old ansatz sometimes gives rise to spurious divergences in the power spectrum, the new ansatz does not!
- But computationally very expensive: cost  $\sim N_g^6$



# If the Bispectrum (or template or decomposition) is factorizable

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$$B(k_1, k_2, k_3) \equiv \sum_i b_1^i(k_1) b_2^i(k_2) b_3^i(k_3)$$

- The old ansatz can be written as a sum of convolution
- Compute convolutions with the help of Fast Fourier Transforms => very significant speed up of the IC generation

$$\Phi_{\mathbf{k}}^{NG} = \frac{1}{6} \sum_i b_1^i(k) \int \frac{d^3 k_2}{(2\pi)^3} G^i(\mathbf{k}_2) Q^i(\mathbf{k} + \mathbf{k}_2)$$

$$G^i(\mathbf{k}) = b_2^i(k) \Phi_{\mathbf{k}}^{*G} / P(k)$$

$$Q^i(\mathbf{k}) = b_3(k) \Phi_{\mathbf{k}}^G / P(k)$$

- The new ansatz may also be factorized

$$\frac{k_1^3 k_2^3 k_3^3}{k^3 + k_1^3 + k_2^3 + k_3^3}$$

**WORK IN PROGRESS**

# Using a smaller grid for $\phi_k^{\text{NG}}$

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- Initial  $\phi_k^{\text{NG}}$  computation scales as  $\sim N_g^6$
- Choose a grid size for  $\phi_k^{\text{NG}}$  of 400 (computation takes 2 days on 256 cores)
- Gaussian grid size is 1024
- Box size 1875 Mpc/h
  - => “NG resolution” 5 Mpc/h  $\sim 3 \times 10^{13} M_{\text{sun}}/h$
- One billion particles per simulation
  - => Particle mass  $\sim 5 \times 10^{11} M_{\text{sun}}/h$
- Evolve simulation with Gadget-2 (takes 1 day on 256 cores)
- Numerical tests confirmed the expected lower mass limit of resolved halos to be  **$3 \times 10^{13} M_{\text{sun}}/h$**

# Simulations

**Table 1.** N-body simulations.  $N_g$  denotes the size of the grid used for the non-Gaussian part of the potential. The size of the particle grid is identical to the size of the grid used for the Gaussian part of the potential and is given by  $N_p$ .

type of non-Gaussianity	$f_{\text{NL}}$	$N_g$	$N_p$	# realizations
local	250	400	1024	1
local	250	1024	1024	2
local	60	1024	1024	3
equilateral template	1000	400	1024	2
equilateral template <sup>a</sup>	1000	1024	1024	1
orthogonal template	-1000	400	1024	2
orthogonal template	-250	400	1024	3
Gaussian	-	-	1024	3

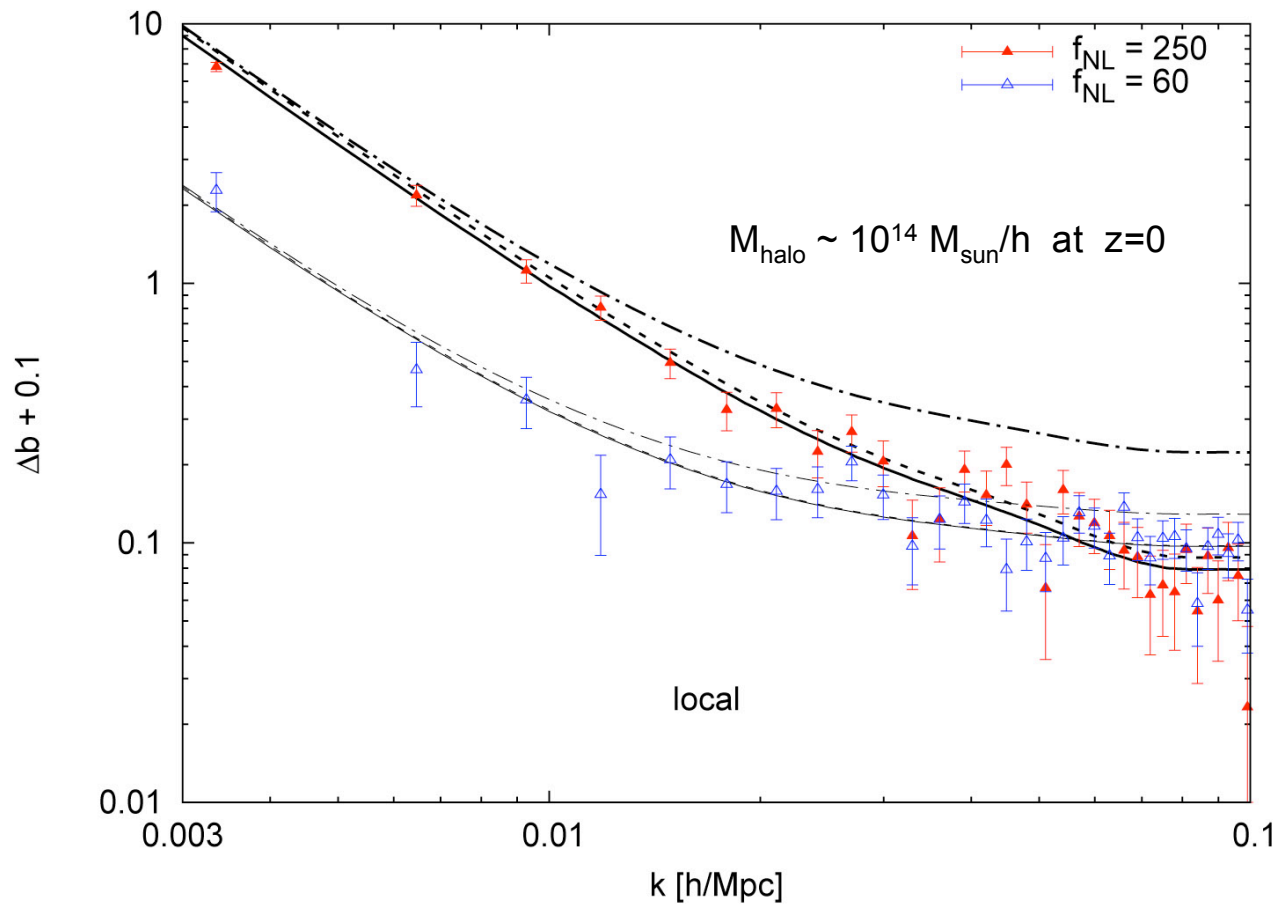
<sup>a</sup> In this case, we use ansatz Eq. (4.5) for the non-Gaussian  $\Phi_{\mathbf{k}}^{NG}$ . The convolution is computed with an FFT in real space.

**Simulation data is (or will be on request) publicly available at:**

<http://icc.ub.edu/~liciaverde/NGSCP.html>

# NG halo bias – local $\sim k^{-3}$

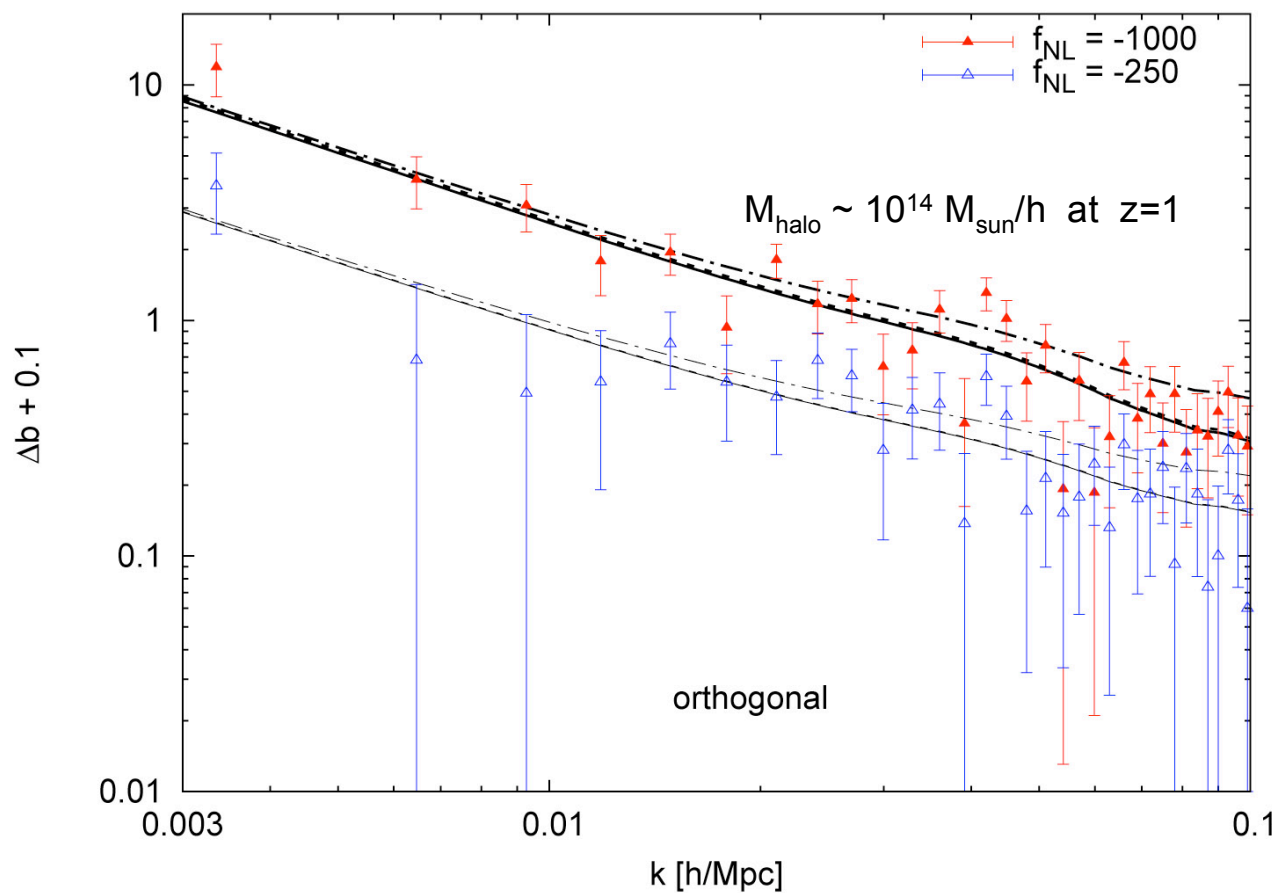
(squeezed limit)



$$\Delta b(k) = \Delta b_I + f_{\text{NL}} [b_1^G + \Delta b_I - 1] \frac{q\delta_c}{D(z)} \frac{\mathcal{F}_M(k)}{\mathcal{M}_M(k)}$$

# NG halo bias – “orthogonal” $\sim k^{-2}$

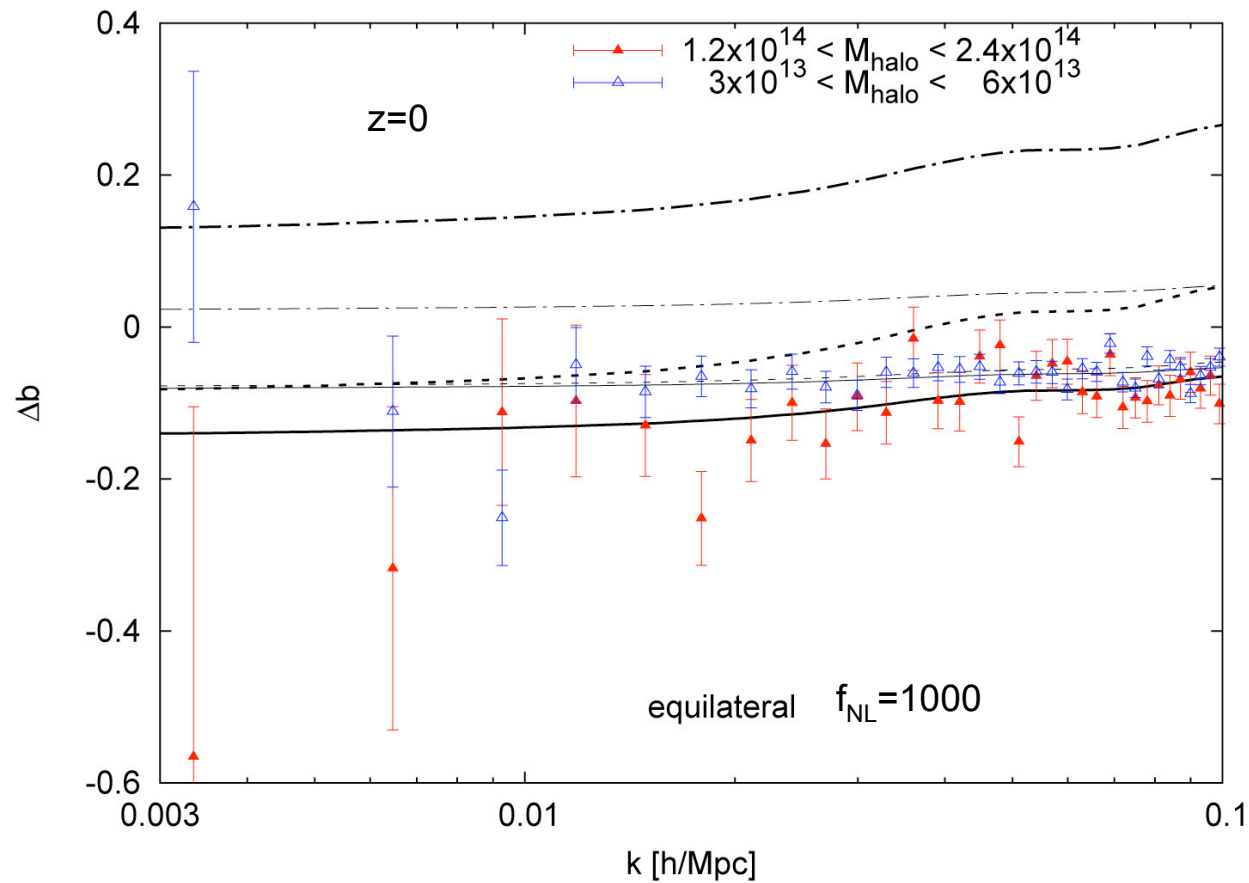
(squeezed limit)



$$\Delta b(k) = \Delta b_I + f_{NL} [b_1^G + \Delta b_I - 1] \frac{q\delta_c}{D(z)} \frac{\mathcal{F}_M(k)}{\mathcal{M}_M(k)}$$

# NG halo bias – equilateral $\sim k^{-1}$

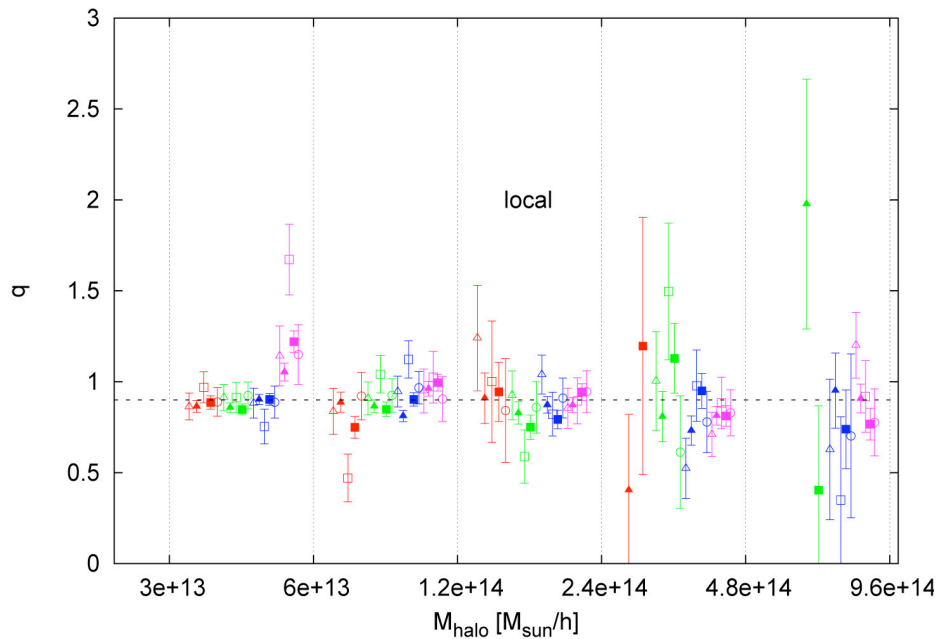
(squeezed limit)



$$\Delta b(k) = \Delta b_I + f_{\text{NL}} [b_1^G + \Delta b_I - 1] \frac{q\delta_c}{D(z)} \frac{\mathcal{F}_M(k)}{\mathcal{M}_M(k)}$$

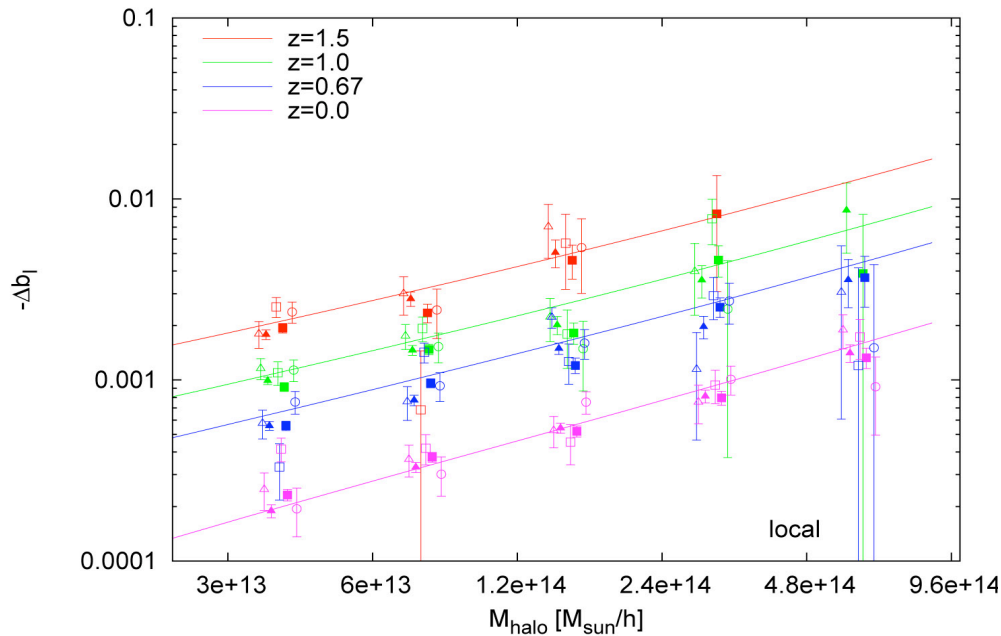


# Fitting results – local $\sim k^{-3}$ (squeezed limit)



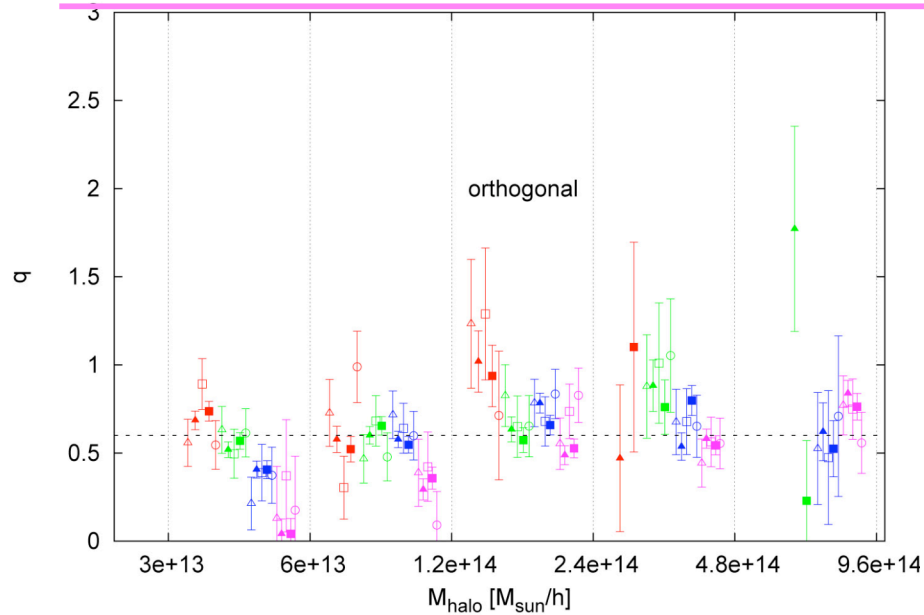
$$\Delta b(k) = \Delta b_I + f_{\text{NL}} [b_1^G + \Delta b_I - 1] \frac{q \delta_c}{D(z)} \frac{\mathcal{F}_M(k)}{\mathcal{M}_M(k)}$$

For the local case, no significant mass or redshift dependence of the fudge factor  $q$  detectable



$$\Delta b_I = b_1^{NG} - b_1^G = - \frac{\partial \ln R^{NG}(M)}{\partial \delta_c}$$

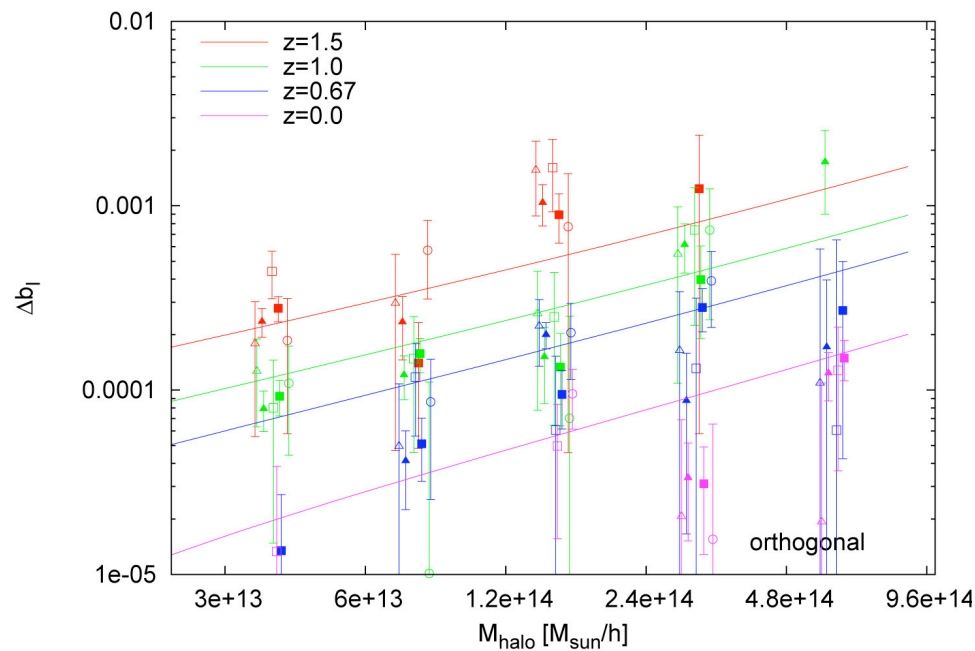
# Fitting results – “orthogonal” $\sim k^{-2}$ (squeezed limit)



$$\Delta b(k) = \Delta b_I + f_{\text{NL}} [b_1^G + \Delta b_I - 1] \frac{q \delta_c}{D(z)} \frac{\mathcal{F}_M(k)}{\mathcal{M}_M(k)}$$

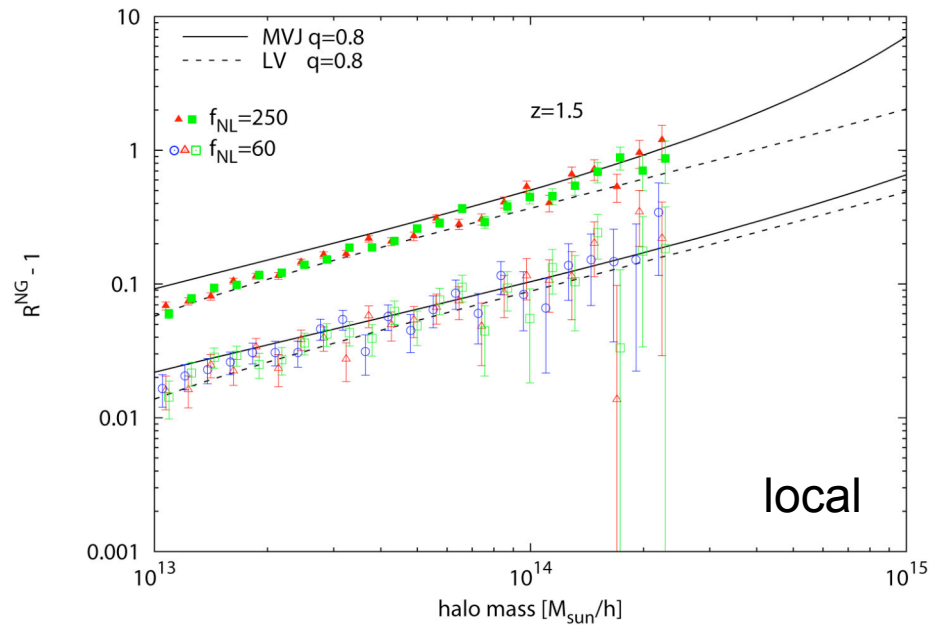
low  $q$ :  $\sim 0.6-0.8$

There are hints for a mass and redshift dependence of the fudge factor  $q$ !

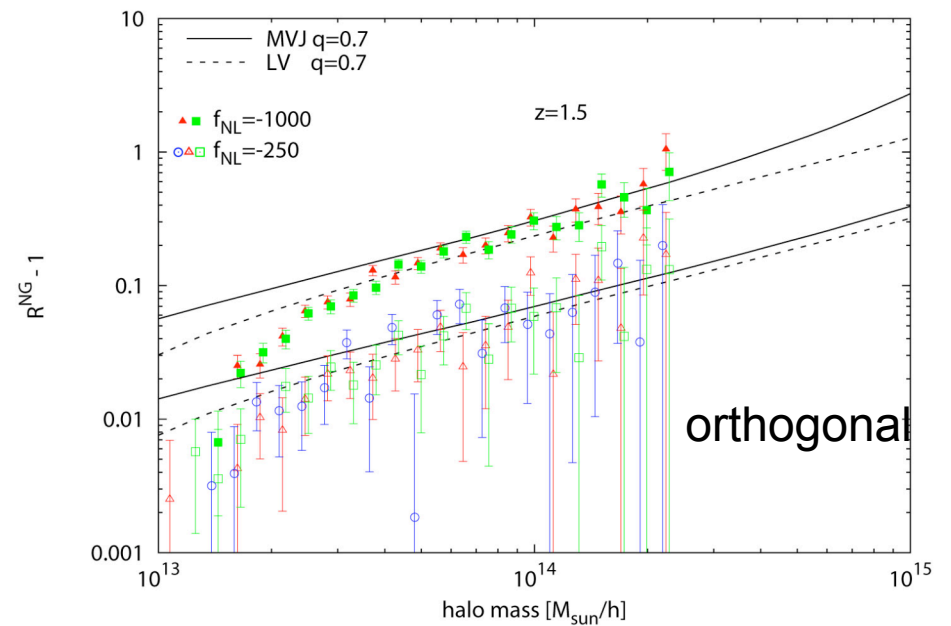
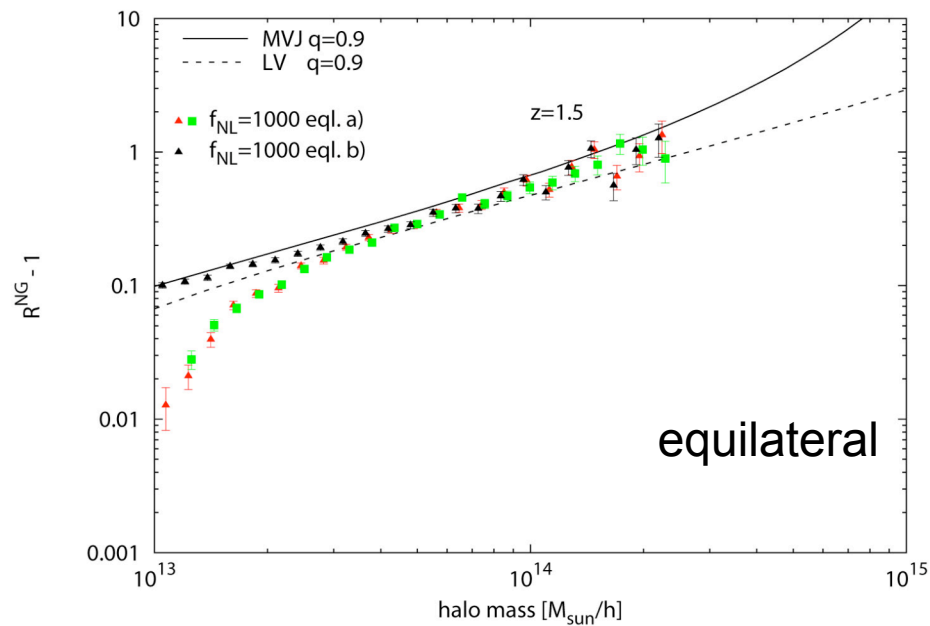


$$\Delta b_I = b_1^{NG} - b_1^G = - \frac{\partial \ln R^{NG}(M)}{\partial \delta_c}$$

# Ratio of Mass function



Theoretical predictions fit the data well, if a shape-dependent fudge factor is taken into account.



# Conclusions from sims

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- Non-Gaussian initial condition for N-body simulations with generic bispectrum possible (but so far computationally expensive)
- N-body results for the (non)-local NG bias are consistent with theoretical predictions after the fudge factor  $q$  is calibrated
- This  $q$ -correction is shape-dependent and for some shapes  $q$  is redshift and mass dependent => **further modeling needed!**
- Interestingly, the fudge factors for the mass function and the halo bias predictions are consistent with each other. Probably more than a coincidence ...
- **Advice:** Halo bias predictions need to be computed from the physical models not from templates

# Putting it all together

Complementarity!

	CMB Bispectrum		Halo bias	
type NG	Planck	(CM)BPol	Euclid	LSST
1 - $\sigma$ errors				
Local	3 <sup>A)</sup>	2 <sup>A)</sup>	1.5 <sup>B)</sup>	0.7 <sup>B)</sup>
Equilateral	25 <sup>C)</sup>	14 <sup>C)</sup>	–	–
Enfolded	010	010	39 <sup>E)</sup>	18 <sup>E)</sup>
# $\sigma$ Detection				
GR	N/A	N/A	1 <sup>E)</sup>	2 <sup>E)</sup>
Secondaries	3 <sup>F)</sup>	5 <sup>F)</sup>	N/A	N/A

YADAV, KOMATSU & WANDELT (2007) B)  
 CARBONE ET AL. (2008) C) BAUMANN ET AL. (2009);  
 SEFUSATTI ET AL. (2009) E) Verde & Matarrese 2009  
 A) F) Mangilli & Verde 2009, Hanson et al. 2009

# Too big, too early?

## XMMUJ2235.3-2557 is not alone

B. Hoyle, R. Jimenez, LV, 2011, Phys.Rev.D

Cluster Name	Redshift	$M_{200}$	$10^{14}M_{\odot}$
'WARPSJ1415.1+3612' +	1.02	3.33 <sup>+2.83</sup> <sub>-1.80</sub>	
'SPT-CLJ2341-5119' *	1.03	5.40 <sup>+2.80</sup> <sub>-2.80</sub>	
'CIJ1415.1+3612' *	1.03	3.40 <sup>+0.60</sup> <sub>-0.50</sub>	
'XLSSJ022403.9-041328' +	1.05	1.66 <sup>+1.15</sup> <sub>-0.38</sub>	
→'SPT-CLJ0546-5345' *	1.06	10.0 <sup>+6.00</sup> <sub>-4.00</sub>	
'SPT-CLJ2342-5411' *	1.08	2.90 <sup>+1.80</sup> <sub>-1.80</sub>	
'RDCSJ0910+5422' +	1.10	6.28 <sup>+3.70</sup> <sub>-3.70</sub>	
'RXJ1053.7+5735(West)' +	1.14	2.00 <sup>+1.00</sup> <sub>-0.70</sub>	
'XLSSJ022303.0043622' +	1.22	1.10 <sup>+0.60</sup> <sub>-0.40</sub>	
'RDCSJ1252.9-2927' +	1.23	2.00 <sup>+0.50</sup> <sub>-0.50</sub>	
'RXJ0849+4452' +	1.26	3.70 <sup>+1.90</sup> <sub>-1.90</sub>	
'RXJ0848+4453' +	1.27	1.80 <sup>+1.20</sup> <sub>-1.20</sub>	
→'XMMUJ2235.3+2557' +	1.39	7.70 <sup>+4.40</sup> <sub>-3.10</sub>	
'XMMXCSJ2215.9-1738' +	1.46	4.10 <sup>+3.40</sup> <sub>-1.70</sub>	
'SXDF-XCLJ0218-0510' +	1.62	0.57 <sup>+0.14</sup> <sub>-0.14</sub>	

These 15 objects  
should not be there

Rare events

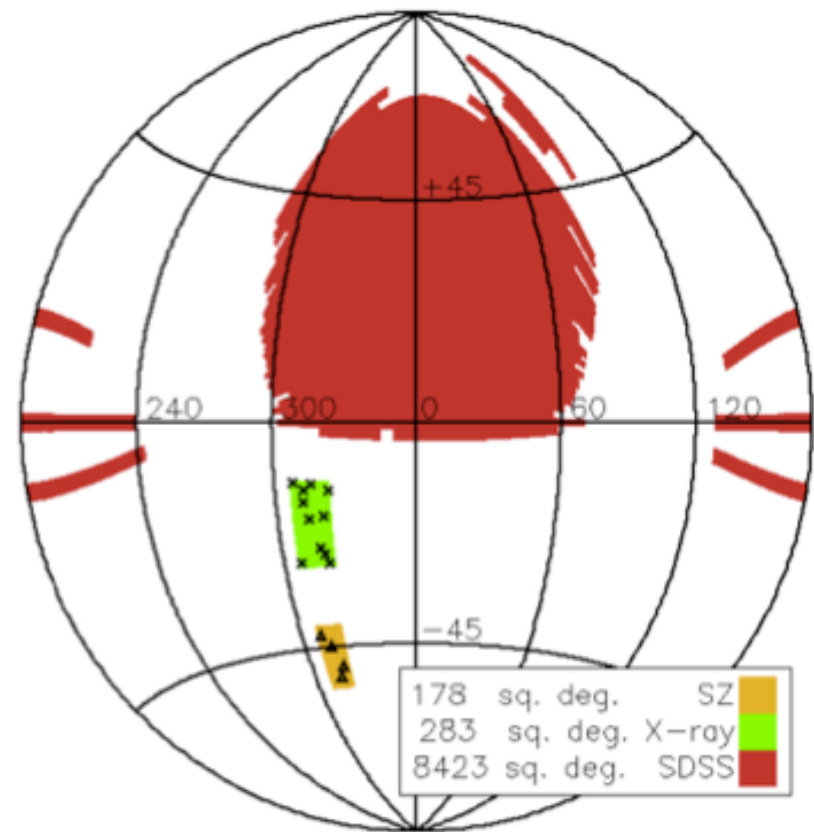
Conservative assumptions about

Mass estimates

Footprint

Survey volume

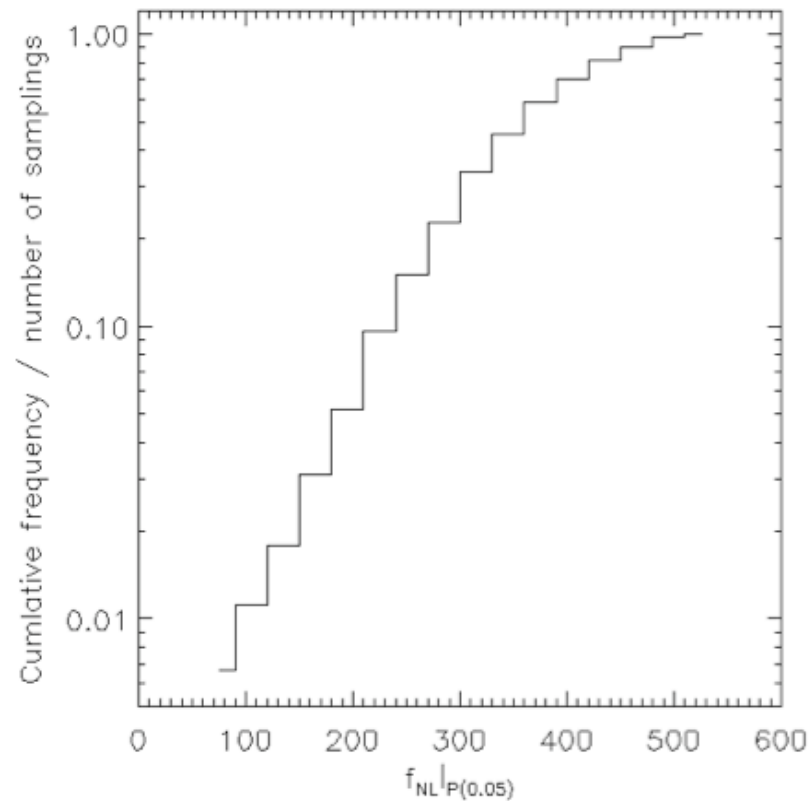
(and mass function adopted)



Rare events

Not even marginalizing over WMAP cosmology...  
Of course, if  $f_{\text{NL}} > 0$  is allowed....

Generalized  
P-values

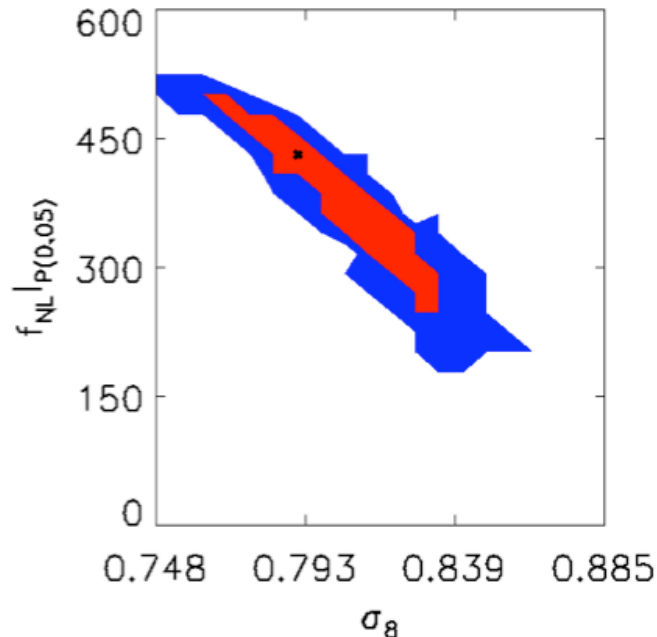


Distribution of the probability that they are allowed at 2-sigmas,  
as function of  $f_{\text{NL}}$

Rare events



# What would one have to do to make $f_{\text{NL}}$ go away?



Say that  $\sigma_8 \simeq 0.90$ .

And accept lower p-values

Such  $\sigma_8$  is  $4 \sigma$  higher than other cosmological probes measures

All cluster masses should have been systematically overestimated by  $1.5 \sigma$

**RELIABLE GRAVITATIONAL LENSES MASSES ARE NEEDED!**  
Submitted HST proposal

Rare events

# Discussion...

